

The Talmud On Transitivity*

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Abstract

Transitivity is a fundamental axiom in Economics that appears in consumer theory, decision under uncertainty, and social choice theory. While the appeal of transitivity is obvious, observed choices sometimes contradict it.

This paper shows that treatments of violations of transitivity already appear in the rabbinical literature, starting with the Mishnah and the Talmud (1st–5th c CE). This literature offers several solutions that are similar to those used in the modern economic literature, as well as some other solutions that may be adopted in modern situations.

We analyze several examples. One where nontransitive relations are acceptable; one where a violation of transitivity leads to problems with extended choice functions; and a third where a nontransitive cycle is deliberately created (to enhance justice).

1 Introduction

Transitivity is one of the most fundamental axioms in Economics. By this axiom, if a supersedes b and b supersedes c , then a supersedes c . This idea is widely used in Economics, for example in consumer theory, decision under uncertainty, and social choice theory.

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This paper discusses violations of transitivity that appear in the rabbinical literature, starting with the MISHNAH and the TALMUD.¹ Our purpose is twofold. We show that modern attitudes to violations of transitivity have a long history and more importantly, we use this ancient literature to offer new perspectives and interpretations of such situations.

There are two main theoretical objections to violations of transitivity. The first is based on ‘Dutch book’ (or ‘money pump’) arguments.² Suppose that a decision maker holds x , and prefers x to y , y to z , but z to x . Offer him first to switch from x to z . As he prefers z to x , he will accept. Then offer him to switch from z to y , which once again he will accept. Finally, for a small commission, offer him to switch from y to x .³ If preferences are continuous, he will switch once more, to be back where he started (holding x), minus the commission he paid. Obviously this behavior is foolish — why pay for nothing?

This argument is relevant when trade is possible. But transitivity is also a key axiom in social choice theory (see e.g. Harsanyi [4] and Sen [16]). The best support for transitivity in this context, or in general choice situations, comes from the requirement that choice should be consistent (see e.g. Sugden [20] or Rubinstein [15]). Let X be a set of alternatives, and let c be a choice function defined on all nonempty subsets of X . Suppose that for three alternatives x, y , and z , the choice function c satisfies $c(x, y) = x$, $c(y, z) = y$, but $c(x, z) = z$. What should $c(x, y, z)$ be? If the outcome of c is a singleton (say x), then there is another outcome that is even better. And if the outcome of $c(x, y, z)$ is not a singleton, then one of these outcomes is dominated by the other.

The Talmud and the rabbinical literature accept the logic of transitivity and use it in many different ways. For example, since reading the book of Esther on Purim (Feast of Lots) postpones temple service, and since temple service is more important than Torah study (Jewish religious studies), the Talmud [22, Meg. 3a] concludes, in line with modern reasoning, that schools are canceled for the reading of the book of Esther.

Sometimes, transitive chains are used by the Talmud in nonstandard ways. When searching for the meaning of a certain biblical phrase, the Talmud may compare it to an identical phrase in a different context, claiming

¹ See Appendix 2 for a glossary and short bios of items in SMALL CAPS fonts.

² See Ramsey [14, p. 78], Tversky [25, p. 45], and Yaari [26, p. 36].

³ Charge the commission only at the last step to avoid the issue of income-dependent preferences.

that both occurrences must have the same meaning. In several cases the Talmud offers a sequence of such citations, each has a common word with its direct predecessor. The last verse then explains the first one.⁴ Another example for a non-trivial application of transitivity is the ruling that if person A owes B money, and B owes C , then A is considered a debtor of C , with the implication that B cannot waive A 's debt.⁵

Because of the wide acceptance of transitive reasoning, the Talmud is sensitive to violations of this rule. In this paper we discuss three examples of such violation. The first is a set of three different activities x , y , and z , for which the rabbis offer reasons why x supersedes y , y supersedes z , and z supersedes x . As this nontransitive cycle does not create any behavioral inconsistencies, the Talmud ignores the theoretical problem, very much like some of the solutions offered in the modern economic literature.

In Section 3 we analyze a problem from the Babylonian Talmud where a choice function defined over two-item cannot extend to include three-item sets. The Talmud offers two solutions. One suggests a violation of the independence of irrelevant alternatives (IIA) axiom. The other is based on ranking *priority rules* rather than options. Solutions that are similar to the former exist in the economic literature, but the latter solution is new, and as we show, can be formalized into a procedural solution to nontransitive cycles.

Our last case (Section 4) is probably the most interesting — it involves medieval interpretations of a difficult Talmudic text, where legal requirements clash with considerations for justice. One of the interpretations suggests preserving a nontransitive cycle in order to force conflicting parties into a compromise. In other words, in such situations nontransitivity is not the problem, but the solution!

Some, if not all, of the examples discussed in this paper may seem contrived. Indeed they are. The way of the Talmud is to create such examples so that it will be able to isolate or examine the fine details of an idea or a rule.⁶ Throughout the paper we use many Hebrew and Aramaic sources. Whenever an English translation exists, we make a reference to it as well. We also add brief descriptions of the different sources cited.

⁴ See [22, Eruv. 51a, Pes. 7b, Ket. 45b,46a].

⁵ See [22, Ket. 19a].

⁶ Moskovitz [12].

2 Priorities

In the Introduction we described a situation where a choice function cannot be extended from choice sets with two items to a choice set with three items. But violations of transitivity may occur even when no choice is involved. As we show below, in such cases the TALMUD and some modern solutions agree that no intervention is needed and a nontransitive cycle can exist without being modified or “solved.” Consider the following observations.

1. Temple services continue on the Sabbath even though the Jewish Law forbids many of these acts (e.g. killing animals) during that day.
2. Executions are not performed on Sabbath.⁷

By means of transitivity, temple service taking precedence over Sabbath and Sabbath over execution imply that temple service should override execution and thus be a refuge to killers. The MEKILTA therefore asserts that scripture is needed to overrule this conclusion, and introduces a third rule

3. Priests are not exempted from death penalty even while performing temple services.⁸

Obviously, this procedure leads to a nontransitive set of rules. But unlike the analysis of the Introduction (and of Section 3 below), where a resolution was needed for the case where the choice set consists of all three element, the cycle we have here relates at any choice node to two options only and no solution is needed for the three-object choice set.

A similar argument is made in the modern literature when nontransitive cycles are detected. Consider the following example by Fishburn and LaValle [2]. A fair die is rolled once and two acts are available:

	1	2	3	4	5	6
f_1	\$1000	\$500	\$600	\$700	\$800	\$900
f_2	\$900	\$1000	\$500	\$600	\$700	\$800

⁷ Rule 1: Num. 28:9–10; [9] Shab., III p. 209; [23] Tzav 2:10, p. 30c; 3:5 p. 31b; Emor 13:7, p. 103d. Rule 2: [18] San. 4:1 and [9] Shab. III p. 209–210.

⁸ [9] Nez. 4, III pp. 37–38 (based on Ex. 21:14).

For example, if the die shows 4, act f_1 yields \$700 while act f_2 yields \$600. The authors argue that although both acts reduce to the same probability distribution, there is nothing wrong in having strict preferences for one act or the other. Some decision makers may prefer f_1 which typically yields a higher outcome than f_2 , while others may prefer f_2 , dreading the sharp disappointment they may feel in case number 2 shows up. But such rationales lead to nontransitive cycles, as by the same logic $f_2 \succ \dots \succ f_6 \succ f_1$ (or $f_2 \prec \dots \prec f_6 \prec f_1$), where f_3, \dots, f_6 are given by

	1	2	3	4	5	6
f_3	\$800	\$900	\$1000	\$500	\$600	\$700
f_4	\$700	\$800	\$900	\$1000	\$500	\$600
f_5	\$600	\$700	\$800	\$900	\$1000	\$500
f_6	\$500	\$600	\$700	\$800	\$900	\$1000

Fishburn and LaValle argue that these preferences do not lead to a Dutch book as the preferences over f_1 and f_2 do not teach us anything about the preferences over $\{f_1, f_2, f_3\}$ (a similar argument is made by Loomes and Sugden [6] and Sugden [21]). When choosing between f_1 and f_2 the decision maker should not concern himself with his possible choice from $\{f_2, f_3\}$ as (within the given context), $\{f_1, f_2, f_3\}$ is not a choice set. Observe that a similar reasoning supports the famous “Rock, Paper, Scissors” game used by children all over the world for a conflict resolution. It works *because* it is applied to two-item sets only.

As stated in the Introduction, the Talmud accepts the logic of transitive deduction. In fact, the Talmud uses transitivity in many nontrivial ways that go beyond the standard “ $x \succ y \succ z$ hence $x \succ z$ ” argument. And yet, logical conclusions based on transitivity, like all other logical conclusions in the Talmud, are not as powerful as their premises. In other words, the fact that x supersedes y and y supersedes z should indeed imply that x supersedes z , but this conclusion is weaker than the first two, and may be rejected by a direct evidence for the opposite, namely z supersedes x .

3 Choice Functions

This section describes a case in which the TALMUD struggles to extend a choice function defined on pairs to a larger set. Three rules define the choice

function over pairs, but these choices cannot be extended to a three item set.

The Talmud in tractate Zevaḥim [22, Zev. 90b] asks in what order should the following animals be sacrificed in the Temple when they are all present: 1. Tithe (from the newborn cattle); 2. A bird for the purification⁹ of a woman after birth-giving; and 3. The lamb for the burnt-offering accompanying the bird.¹⁰ Three rules used by the Talmud for pairwise choices lead to a nontransitive cycle.

Rule 1 Consumption of the flesh. Among cattle offerings, burnt-offering, which is wholly consumed by fire on the altar precedes sacrifices that are partially eaten, like tithe.

Rule 2 Type of animal. Cattle precede birds, hence cattle burnt-offerings and tithe should precede bird purification-offerings.

Rule 3 Type of sacrifice. A purification-offering precedes the burnt-offering accompanying it, even when the purification-offering is a bird and the burnt-offering is from the cattle.¹¹

The Talmud deliberates: “Which of these precede? Shall the bird purification-offering come first? there is tithe. Shall tithe come first? there is the cattle burnt-offering. Shall the burnt-offering come first? there is the bird purification-offering that must precede it!” [22, Zev. 90b].

This is the problem mentioned in the Introduction — how to extend a choice function that is defined on pairs to larger sets. Consider the three items bird purification-offering (henceforth “bird”); the lamb burnt-offering adjoining it (“lamb”); tithe (which can be either calf or lamb), and let c be a choice function. By *Rule 2* we have $c(\text{bird, tithe})=\text{tithe}$; by *Rule 1*,

⁹Following Milgrom [10, pp. 758–9], we use here the term purification-offering rather than sin-offering used in the King James Bible.

¹⁰Tithe: Lev. 27:32 and [18] Zev. 5:7–8. Birth-giving woman: Lev. 12:6 and RASHI [22] Zev. 90b s.v. *Ayla bah*. The wording of the Talmud is general and seems to refer to any purification- and burnt-offerings (this is stated explicitly in a later layer of the Talmud — see [22], Hor. 13a). But according to the SIFRA, where Rule 3 below first appears, this rule applies only in cases where the two offerings constitute a closed pair, and the only case of a bird-lamb combination is the birth-giving woman (see Minkowski [11, p. 104]).

¹¹Rule 1: [18] Zev. 10:2. Rule 2: *ibid*, also Rashi [22] Zev. 89a s.v. *Mipney shehu*. Rule 3: [19] Ḥova 18:5–6, p. 189. Note the different sources of the three rules — Rules 1 and 2 are from the Mishna, while Rule 3 is from the Sifra. Actually, the Talmudic passage discussed here is an attempt to reconcile the apparent disagreement between these sources.

$c(\text{lamb, tithe})=\text{lamb}$; by *Rule 3*, $c(\text{bird, lamb})=\text{bird}$; and the question posed by the Talmud is what is $c(\text{tithe, bird, lamb})$? Two answers are offered.

1. “Here (in Babylon) they explained that cattle precede birds” (Talmud text) and therefore “tithe takes precedence” (Rashi). In other words, the order at which the three offerings are sacrificed is tithe, then the bird, and finally the lamb.
2. “The sages of the west (i.e., Palestine) say: The superiority of the cattle burnt-offering over tithe serves (lit.: enters into) the bird purification-offering and elevates it over the tithe.” According to this solution, the order is bird, lamb, and finally tithe.

The justification for the Palestinian rule is surprisingly similar to modern arguments. It suggests that although when the two stand against each other, tithe precedes any bird offering, at the presence of the burnt-offering that adjoins the purification-offering, this order is reversed, and now the bird precedes the tithe. This is an obvious violation of the Independence of Irrelevant Alternatives axiom:

IIA If T is a subset of S and $c(S)$ is in T , then $c(T) = c(S)$.

The rationale for IIA, like the rationale for the Weak Axiom of Revealed Preference,¹² is that the ranking of any two outcomes should not depend on the availability of other possible outcomes. This is exactly the intuition rejected by the Palestinian solution. Although it is true that when the choice is between tithe and a bird tithe takes precedence, the existence of the third option, the burnt-offering, changes the nature of the bird purification-offering, and now the pair {bird,lamb} is elevated above the tithe.

In a famous example, Luce and Raiffa [7, p. 288] tell the story of a diner in an unfamiliar seemingly modest restaurant. There is no menu, but the waiter tells him that he may have either broiled salmon or a steak. After his choice of salmon is reported to the kitchen, the waiter tells him that in fact they also have fried snails and frog’s legs. The diner detests both, but now wants to order the steak instead of the salmon. The explanation offered by Luce and Raiffa is simple. Initially, the traveler was uncertain about the quality of the restaurant, and as he prefers a mediocre salmon to a mediocre steak, he

¹² WARP is a necessary and sufficient condition of the rationalization of a choice function through a transitive preference relation, see e.g. [15].

ordered the former. The fact that the restaurant serves fancy food tells him that his choice is actually between a high quality salmon and a high quality steak, of which he prefers the latter. The existence of the other alternatives, even if not chosen, changes the nature of the chosen options. According to the sages of the west, something similar happens here. Although it is generally true that the tithe precedes bird offerings, the existence of the burnt-offering lamb that adjoins the birth-giver’s bird purification-offering enhances this bird offering and elevates it above the tithe.

The Babylonian solution requires some explanation. We mentioned above three rules: consumption of the flesh, type of animal, and type of sacrifice. The dictate “cattle precede birds” is a paraphrase of the MISHNAH [18, Zev. 10:3] explanation for why the tithe precedes the bird-offerings, but it does not yield a complete order, as tithe–lamb–bird, tithe–bird–lamb, and lamb–tithe–bird are all consistent with it. Another rule must therefore have been previously applied. As is explained by Rashi,¹³ the third rule dominates the other two since it is directly derived from Scripture, hence the purification-offering must be sacrificed before the burnt-offering. After the (implicit) application of this rule, the Babylonian solution is to use the second rule (type of animal), which now implies that tithe precedes the bird. These two rules, in that order, yield tithe before bird and the latter before the lamb, which, by transitivity, also implies tithe before that lamb.

Kalai, Rubinstein, and Spiegel [5] introduced the concept of *rationalization by multiple rationales*, where the set of strict preferences \succ_1, \dots, \succ_k over a set S thus rationalize a choice function if for every $T \subseteq S$ there is \succ_i such that $c(T)$ is \succ_i -maximal in T . The Babylonian solution does that and goes a step further by ranking the partial preferences $\{\succ_i\}$. The rules are applied one at a time until a complete and transitive order is created. Naturally, not all rules are used. We formalize this idea in Appendix 1.

The idea that rules should be ordered is both natural and normatively appealing. Many judicial decisions involve conflicting past rulings or legislations and therefore require courts and legislators to rank rules (e.g., the superiority of the constitution over other laws or the superiority of the supreme court over other courts). Individuals too may rank some of their decision rules higher than others. For example, ranking potential real estate by location, size, and price may lead to conflicting orderings, yet ranking the criteria may help resolve these conflicts.

¹³ [22] Zev. 90b, s.v. *Min zevah adif*.

4 Cycles and Fairness

This section differs from the rest of the paper and from conventional wisdom about the negative aspects of nontransitive cycles. It describes a case where a nontransitive cycle can be eliminated, but is nevertheless welcomed and actually enforced in order to promote justice.

According to the Jewish Law, when a man marries a woman he signs a contract (“KETUBAH”) which lists properties she will receive upon his death or when she is divorced. In the meanwhile, the rights and fruits of these properties belong to the husband who may sell them, with the explicit understanding that if he dies or divorces his wife, the properties are to be transferred to her. (This condition is presumably reflected by the price of the property). A man may marry more than one wife, and if he has only one property, the same property is assigned to all of them. The first wife has precedence over the second, but if she dies before their husband, the second wife inherits the property.

Suppose a man married two women and then sold his only field. The order of claims is simple enough, and is fully determined by the order of events. The first wife has the strongest claim to the field, then the second wife, and the buyer has the weakest claim. The MISHNAH [18, Ket. 10:5] complicates matters by adding a twist in which the first wife gives a written declaration to the buyer, saying ‘I have no claim whatsoever upon you.’ The Mishnah rules that upon their husband’s death, “the second wife may distraint on the buyer, and the first wife on the second, and the buyer on the first wife, and so they go on in turn until they arrange some compromise between them.”¹⁴

The cycle results from the fact that although the first wife waived her rights to seize the field from the buyer, she did not forego her claims against the second wife. The latter can claim the field from the buyer, as she married her late husband *before* the field was sold. After the field is transferred to the first wife, the written declaration signed by her enables the buyer to take possession of the field, thus creating a cycle.

The TALMUD [22, Ket. 95b] analyzes another invented case which is some-

¹⁴ The Mishnah is usually not shy of dictating specific solutions, as is done for example in an earlier Mishnah in the same chapter, where exact divisions are laid out for various scenarios (see analysis in Aumann and Maschler [1]). Since nothing is said here about the nature of the compromise, it seems that the Mishnah deliberately refrains from doing so. Medieval commentators offer all possible combinations: between all three, any set of two or specific sets of two.

what similar to the story of the Mishnah, in which a different ruling is obtained. In this case, a man said to a married woman ‘My property shall be yours and after you it shall be given to person X .’ The woman then sold the property and died. Her husband may now seize the property from the buyer because his wife received the property after their marriage and therefore he ‘bought’ it before the buyer.¹⁵ The woman’s successor, person X , seizes it from the husband because, as explained by the Talmud, the original owner meant to tell the woman ‘After your death, only your successor shall acquire possession; your husband shall not.’ Now the buyer can seize the property back from the successor because he bought it while the woman was still alive, when the successor did not yet have a claim for the property. Abbaye (Babylon, mid. 4th century) rules that the property should be confirmed in the possession of the buyer. This seems problematic. As all parties appear to have the same power, why should the property stay at the hands of the buyer? Moreover, as is asked by the Talmud, “why should this case be different from the Mishnah where we learned ‘and so they go on in turn until they arrange some compromise between them’?” The Talmud offers the following surprising answer: “In the Mishnah they all gave up something. But here it is only the buyer who paid.” [22, Ket. 95b].

In the Mishnah, each of the three agents (buyer, first, and second wife) gave up something for the disputed asset. The buyer obviously paid for it, and the two women entered into a contract with their late husband, in which they gave him the right to use some of their assets in return for rights to the field after his death. But in Abbaye’s story of the buyer, husband, and successor, it is only the buyer who paid anything and he is the only one to suffer a loss. The husband was never intended to receive anything, and for the successor too it was like a windfall. It is therefore just to leave the property in the hands of the buyer.

The crux of Abbaye’s ruling is that the situation of the three parties (buyer, husband, and successor) are not symmetric and therefore they should receive differential treatment. A similar attitude appears in the social choice literature. In Condorcet’s voting paradox three voters α , β , and γ vote on three issues A , B , and C . Their rankings are ABC , BCA , and CAB , hence two out of three prefer A to B , B to C , and C to A and majority rule leads to a nontransitive cycle.

¹⁵ Marriage is considered an implicit sale (to the husband) of all the properties a woman acquires during the tenure of the couple joint life. See [18, Ket. 8:1], [22, Ket. 50a].

This procedure ignores possible differences in the intensity of preferences. Social welfare functions solve this paradox by maximizing a function of individual utilities, for example, by taking a (weighted) sum of these utilities (Harsanyi [3, 4]). If the utilities of α from the three options are 10, 8, 6, the utilities of β are 6, 10, 8, and the utilities of γ are 8, 6, 20, then the sum of utilities is maximized at option C , and society prefers this option to the other two (between which it is indifferent). Utilitarianism breaks the spell of infinite cycles by paying attention to asymmetries between voters. Abbaye is doing something similar here — the situation is not symmetric because only the buyer suffers a real loss.

But how can the legal infinite cycle be stopped? RASHI offers the following explanation. The first time the husband approaches the buyer, he has a just claim for the property. The buyer obtained it from the husband's late wife and he, the husband, "bought" it first. With respect to the woman's assets, the husband has a priority over the buyer. But after the buyer seizes the property from the successor the court can refuse the husband's claim as now the buyer holds an property that was taken from someone else and not from the wife. With respect to assets taken from the successor the husband's hand is inferior to the buyer's, as the buyer can seize properties from the successor while the husband cannot.¹⁶

Rashi shows how to stop after one round — but exactly the same distinction between the first and second round exists in the Mishnah! When the second wife first approached the buyer, her claim was based on the fact that she implicitly bought the field from her husband before the field was sold to the buyer. But in the second round the buyer holds the same field after receiving it from the first wife and not from the husband. Rashi's explanation should lead us to the conclusion that the field should stay at the hands of the buyer, but the Mishnah orders the cycle to continue until a compromise is reached.

This observation forces us to understand the Talmud according to Rashi in the following way. There is an infinite nontransitive cycle which occurs when a certain person is changing the default rules (the first wife in the Mishnah, the original owner in the story of Abbaye). Although there is a legal way to break the resulting infinite cycle, the Mishnah prefers not to use it, but

¹⁶ [22, Ket. 95b] s.v. *Umuqminan*. Rashi's language is somewhat ambiguous regarding the actual existence of a first round. But the Tosafist R. Isaiah di Trani (Italy, d. circ. 1240), essentially offering Rashi's interpretation and wording, explicitly suggests one round. See also a remark at the end of this section.

rather to keep the cycle going. Not only does the Mishnah not stop the cycle, it actually rules that it must go on until the parties compromise. According to the Talmud, both rulings, this of the Mishnah and that of Abbaye, follow justice considerations. Abbaye used the legal distinction to do justice to the buyer. But in the Mishnah, where all parties are equal, it would be unjust to leave the field in the hands of the buyer. A just solution that takes care of the interests of all parties is preferred to the legal loophole that can stop the cycle.¹⁷ Typically, infinite cycles are the problem, but here such cycles serve as a solution to an unjust allocation. The idea that compromises may solve a locked political situation is obvious. The idea that a nontransitive cycle is deliberately kept in order to force the disputing parties into a compromise is, to the best of our knowledge, novel.

Remark: The above analysis leaves one question open — why does Abbaye bother the courts into a redundant cycle which eventually leaves the property at the hands of the current holder? This question led the Tosafist R. Samson (“Rash” of Sens, France, d. 1230)¹⁸ to conclude that the property is never seized from the buyer and Abbaye’s words should be understood as theoretical thoughts (“since the husband *may* seize...the property is confirmed in the possession of the buyer”).¹⁹ The Economics literature offers a somewhat similar reaction to Dutch books. Such mechanisms are based on the assumption that decision makers agree to move in cycles, something they are not likely to do (see Fishburn and LaValle [2]).

5 A Tale of Two Cycles

CASE 1: Sometime around the year 1000 CE, a question [13, §175] was sent to R. Ḥanoḥ (b. Moshe, of Spain, d. 1025). Jacob²⁰ married Leah and they

¹⁷ We want to emphasize that Rashi does not explicitly offer this interpretation of the Mishnah — in fact, he offers no commentary to that part of the Mishnah at all. But our suggestion seems to be the only possible interpretation following Rashi’s comments.

¹⁸ See Tosafot [22, Ket. 95b s.v. *Umuqminan*], [24, p. 272]. See also Maimonides [8, Zechiya Umatana 12:12].

¹⁹ This conclusion probably requires a different legal break of the infinite cycle, which is offered by the TOSAFOT in [22, BB 139b s.v. *Hatam*]: Abbaye considers the husband an heir and not a buyer, therefore he cannot seize the property from the buyer even in the first round. Observe that this loophole cannot be used in the Mishnah where both wives are considered “buyers.”

²⁰ As is common in the Halakhic literature, generic names are used in actual cases.

had several daughters. Later he also married Rachel. Jacob then sold the fields brought to him by Rachel, and gave her instead a bill on one of his own fields. Later, when Leah's daughter married, Jacob, with the full consent of Leah, gave her the field he pledged to Rachel. He died, and now Rachel wants to seize the fields from Leah's son-in-law, as unlike Leah, she never agreed to this present.

R. Ḥanoch analyzes this case by using the MISHNAH of Ketubot (see Section 4 above), and concludes that Rachel can indeed seize the field, but will not be able to keep it against a claim by Leah, as the latter married Jacob first. Leah too will not be able to keep the fields but will have to surrender them to her daughter. This cycle should continue until they reach a compromise.

CASE 2: About 300 years later, a somewhat similar question [17, 63:4] was sent to R. Asher (b. Yeḥiel, "Rosh," Germany and Spain, d. 1327). A couple sold a house and wrote to the buyer that they do not leave themselves any rights to this property. The buyer finds out, after the death of the husband, that the house was mortgaged by the sellers against a loan, and the lender now wants to seize the house from the buyer. The problem is that the woman who (with her late husband) sold the house shows her KETUBAH, which is earlier to the loan, and informs the lender that she will seize the house from him and transfer it back to the buyer. Citing the above Mishnah, R. Asher ordered the house to remain in the hands of the buyer.

How could two courts reach such different conclusions from the same Mishnah? The difference is in the (implicit) coalition structure of the two stories. In the case of R. Ḥanoch, each of the parties would like to keep the fields. This is obviously true for Rachel and Leah's daughters, but is also the case with Leah, who may want her daughters to prosper, but will not necessarily like to transfer to her sons-in-law assets that are supposed to support her through her years of widowhood. Like the conflict of the Mishnah, the three parties have to reach a compromise.

The story of R. Asher is different, as the remaining seller makes it clear that she intends to transfer the house back to the buyer. In his answer, R. Asher considers this to be the compromise ordered by the Mishnah and concludes that the buyer now occupies two positions: that of a buyer, but also that of the woman whose Ketubah dates earlier to the loan. Although R. Asher does not say so, his argument is in line with the Palestinian solution to the sacrifices cycle (see Section 3 above). The Ketubah of the seller elevates

the buyer above the lender, similarly to the way in which the superiority of the cattle burnt-offering over tithe elevates the bird purification-offering over the tithe.

Even if not formal, this may be the most practical solution to nontransitive cycles. We should check for natural connections between elements of the set of options, thus reducing the size of this set. In Section 3, there is an obvious connection between the burnt-offering and the purification-offering, as both are brought, as a pair, by the birth-giving woman. Likewise, the female seller and the buyer of Case 2 above form a natural pair (the seller wants to size the house so that she will be able to transfer it to the buyer, as was her initial intention according to the letter she and her husband wrote to him). When such natural pairs do not exist, other solutions should be used, like prioritizing rules or prioritizing options. Sometimes, it is even better not to offer any solution.

Appendix 1

This appendix explains how to create a complete ranking from ordered pairwise comparisons.

Let $N = \{1, \dots, n\}$ and let c be a choice function such that for all $i \neq j$, $c(i, j)$ exists and is a singleton. Let R be a linear ranking of the $n(n-1)/2$ pairs $\{i, j\}$, $i \neq j$, that is,

$$\{i_1, j_1\} R \{i_2, j_2\} R \dots R \{i_{n(n-1)/2}, j_{n(n-1)/2}\}$$

and for no $k > 1$, $\{i_k, j_k\} R \{i_{k-1}, j_{k-1}\}$. Let $\emptyset \neq T \subseteq N$ and construct inductively partial linear orders $\succ_0^T, \dots, \succ_{n(n-1)/2}^T$ on T as follows.

1. $\succ_0^T = \emptyset$ (that is, no elements of T are compared by \succ_0^T). Set $k = 1$ and move to the next step.
2. If $k = \frac{n(n-1)}{2} + 1$ set $\succ^T = \succ_{n(n-1)/2}^T$ and the construction is complete. Otherwise, move to the next step.
3. If $\{i_k, j_k\} \not\subseteq T$, set $\succ_k^T = \succ_{k-1}^T$, increase the value of k by 1, and move to step 2. Otherwise, move to the next step.
4. If i_k and j_k are comparable by \succ_{k-1}^T , set $\succ_k^T = \succ_{k-1}^T$,²¹ increase the value of k by 1, and move to step 2. Otherwise, move to the next step.

²¹ Even if $c(i_k, j_k)$ disagrees with \succ_{k-1}^T . In such a case c is overruled by \succ_{k-1}^T .

5. Define \succ_k^T as follows:

- For all $\{i, j\} \neq \{i_k, j_k\}$, $i \succ_k^T j$ iff $i \succ_{k-1}^T j$.
- If $c(i_k, j_k) = i_k$, then $i_k \succ_k^T j_k$. If $c(i_k, j_k) = j_k$, then $j_k \succ_k^T i_k$.

And let \succ_k^T be the transitive closure of \succ_k^T . Increase k by 1 and move to step 2.

By construction, the ranking \succ^T is transitive. It is also complete. Let $i, j \in T$ and let k such that $\{i_k, j_k\} = \{i, j\}$. Either i_k and j_k were comparable by \succ_{k-1}^T , and then they are also comparable by \succ_k^T , or \succ_k^T is constructed to compare them. As $\succ_1^T \subseteq \dots \subseteq \succ_{n(n-1)/2}^T = \succ^T$, the ranking \succ^T is complete.

This method may not work if we drop the requirement that for no $k > 1$, $\{i_k, j_k\} R \{i_{k-1}, j_{k-1}\}$. Let $N = \{a_1, \dots, a_4\}$, and suppose that $c(a_1, a_2) = a_1$, $c(a_2, a_3) = a_2$, $c(a_3, a_4) = a_3$, and $c(a_1, a_4) = a_4$. Suppose further that the top four pairs are $\{a_1, a_2\} R \{a_3, a_4\} R \{a_2, a_3\} R \{a_1, a_4\}$ but also $\{a_1, a_4\} R \{a_2, a_3\}$. the two orders $a_1 \succ a_2 \succ a_3 \succ a_4$ and $a_3 \succ a_4 \succ a_1 \succ a_2$ are consistent with the above algorithm.

Appendix 2

KETUBAH The Jewish marriage contract. The ketubah lists the dowry items given by the wife's family as well as the husband's obligations and suspended debts to his wife, which she may collect if he dies before her or in a case of divorce. From the date the ketubah is signed, the whole property of the husband (primarily his real estate) is pledged to the contract, and the widow, or divorcee, can seize any part of it from her husband, his heirs, or anyone else, including other wives of her husband, who got holding on the property after that date.

MISHNAH A collection of old rabbinic oral traditions and instructions edited circ. 220 CE. The Mishnah is composed of sixty tractates, ordered in six "order" (sedarim). Each single paragraph of the Mishnah is also called a Mishnah.

MEKILTA The early (3rd c CE) rabbinic commentary to Exodus, attributed to the school of R. Ishmael.

RASHI R. Shlomo b. Yitzhak (d. 1105), preeminent commentator to most parts of the Bible and the Babylonian Talmud. In this article, reference is made to Rashi’s commentary printed on the inner margins of the traditional editions of the Babylonian Talmud.

SIFRA The old (3rd c CE) rabbinic commentary to Leviticus, attributed to the school of R. Akiva.

TALMUD Babylonian and Palestinian. The collection of Rabbinic materials of various genres — rulings, discussions, interpretations, stories, morals, and other literary forms — following the order of the Mishnah. We deal here only with passages from the Babylonian Talmud (3rd-5th c CE).

TOSAFOT A fragmentary, critical commentary to the Babylonian Talmud, by and large the product of the French and German Talmudic schools of 12th–13th centuries. In this article, reference is made to the Toasafot printed on the outer margins of the traditional editions of the Babylonian Talmud.

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